Day 2 - AM

<u>Prove</u>: $\lim_{x \to c} f(x) = k$ if $f(x) = k$

<u>Proof</u>: let $\varepsilon > 0$. Choose any $\delta > 0$. Assume $|x-c| < \delta$
then $|f(x) - k| = |k - k| = 0 < \varepsilon$. ∎

<u>Average Rate of Change</u> (AROC) from a to b is
$$\frac{f(b) - f(a)}{b - a}$$

<u>Instantaneous Rate of Change</u> (IROC) at a is
$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = f'(a)$$

this is also called the <u>derivative</u>.

<u>L'Hopital's Rule</u>

Consider $\displaystyle \lim_{x \to c} \frac{f(x)}{g(x)} = \frac{\lim_{x \to c} f(x)}{\lim_{x \to c} g(x)}$. If $\frac{\infty}{\infty}$ or $\frac{0}{0}$

then $\displaystyle \lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \frac{f'(x)}{g'(x)}$.

<u>Partial Derivatives</u> - rates of change with respect to a specific variable.

→

Ex: Compute the partial derivatives of

$f(x,y) = x^2 y^5$

$$\frac{df}{dx} = \frac{d}{dx}(x^2y^5) = y^5(2x) = 2xy^5$$

"y^5 is a constant"

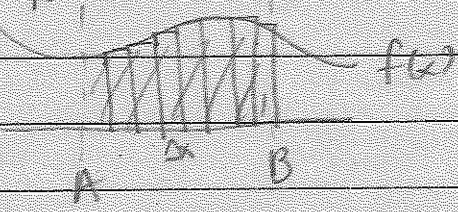
$$\frac{df}{dy} = \frac{d}{dy}(x^2y^5) = x^2(5y^4) = 5x^2y^4$$

"x^2 is a constant"

* See handout on website or back of calculus text for list of derivatives

Integration $\rightarrow$ must be a continuous function over the interval!

$$\int_A^B f(x)\,dx$$



Area under the curve!

$$\text{Area} = \lim_{N \to \infty} \left(\sum_{n=1}^{N} f(x_i)\,\Delta x \right)$$

Fundamental Thm of Calculus

if f is continuous on $[a,b]$ and F is the antiderivative of f on $[a,b]$ then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

if f is continuous on (a,b), define $F(x) = \int_a^x f(t)\,dt$

then $F'(x) = f(x)$

Power Rule $\int x^n\,dx = \frac{1}{n+1} x^{n+1}$

Ex: $\int \sin u\,du = -\cos u + C$

$\int \cos u\,du = \sin u + C$

$\int \frac{1}{x}\,dx = \ln(x) + C$

$\int e^x\,dx = e^x + C$

* find online or in back of textbook a list of integrals

$\longrightarrow$

U-Substitution

Ex: $\int_2^4 x^2 e^{x^3}\,dx$

let $u = x^3$

$\frac{du}{dx} = 3x^2 \quad du = 3x^2 dx$

$= \frac{1}{3}\int_2^4 3x^2 e^{x^3}\,dx$

if $x = 2 \quad u = 8$

$x = 4 \quad u = 64$

$= \frac{1}{3}\int_8^{64} e^u\,du$

$= \frac{1}{3}\left(e^u\right)\Big|_8^{64} = \boxed{\frac{1}{3}\left(e^{64} - e^8\right)}$

Integration by Parts

$$\int f(x)\,g'(x)\,dx = f(x)\,g(x) - \int f'(x)\,g(x)\,dx$$